

1 RADIATION

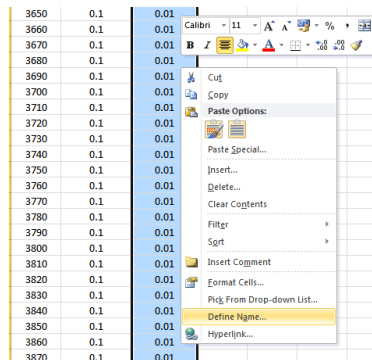
1.1 Determine the photon flux, power density and spectral irradiance for the AM1.5 spectrum. [use spreadsheet and excel data file]

The data in the excel sheet gives data in $[I] = W / m^2 \cdot 10nm$, that is, every data point gives the total irradiance W / m^2 in the interval of $10nm$.

The units of power density, i.e. $I(W / m^2)$. In this case, we have to sum the spectral power density;

$$= \int_0^{\infty} I(\lambda) d\lambda$$

Looking at the data, probably the first thing to do is to take the **power density** $[I] = W / m^2 \cdot 10nm$ data and divide it by 10 to obtain the data as $[I] = W / m^2 nm$. Let us call this data I_{λ} in the excel spread sheet. You can do this by selecting the data (third column) and "define name".



You are now in a position to plot your data - see the plot chart called $I(\lambda)$.

To obtain the total power, or irradiance we now need to perform the relevant summation.

$$I = \sum_{\lambda=280nm}^{\lambda=4000nm} I(\lambda) \Delta\lambda$$

$$= \Delta\lambda \sum_{\lambda=280nm}^{\lambda=4000nm} I(\lambda)$$

It should be obvious that $\Delta\lambda = 10nm$ and remains unchanged. In the excel sheet the $\Delta\lambda$ is defined as delta_lamda .

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The photon flux can be calculated by:

$$\phi(\lambda) = \frac{I(\lambda)}{E_{\text{photon}}(\lambda)}$$

You can check that this makes sense by looking at the units, i.e. the photon flux is per unit of area, per unit of time, per unit of wavelength:

$$[\phi(\lambda)] = \frac{[I(\lambda)]}{[E_{\text{photon}}(\lambda)]} = \frac{W/m^2 \cdot \lambda}{J} = \frac{J/s \cdot m^2 \cdot \lambda}{J} = \frac{1}{m^2 \cdot s \cdot \lambda}$$

Going to our excel sheet we first calculate the photon energy $E_{\text{photon}}(\lambda)$ [variable E_photon] and to then calculate the photon flux $\phi(\lambda)$ [variable Phi].

The photon energy is given by:

$$E_{\text{photon}}(\lambda) = h\nu = \frac{hc}{\lambda}$$

Again, in our case, because the data is discrete:

$$\phi_{\lambda}(\lambda) = \frac{I_{\lambda}(\lambda)}{E_{\text{photon}}(\lambda)}$$
$$[\phi_{\lambda}(\lambda)] = \frac{1}{m^2 \cdot s \cdot \lambda}$$

The data is plotted in Phi(lambda).

1.2 Use the PVGIS tool to determine the increase in yield using monthly inclination adjustment [reference: fixed system with optimum inclination for location; use insolation data for your hometown]

First visit the PVGIS website to find the tool. Below you can find an example where firstly the option to let the slope be optimised automatically was selected.

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The screenshot shows the PVGIS web application interface. The left panel displays a map of the region around Amadora, Portugal, with a red pin indicating the selected location. The center panel is titled "Performance of Grid-connected PV" and contains the following configuration options:

- Radiation database: Climate-SAF PVGIS
- PV technology: Crystalline silicon
- Installed peak PV power: 1 kWp
- Estimated system losses: 0%
- Mounting position: Free-standing
- Slope: 34°
- Azimuth: 0°
- Tracking options: Vertical axis, Inclined axis, 2-axis tracking (all unchecked)
- Output options: Show graphs, Show horizon, Web page, Text file, PDF (all unchecked)

The right panel displays the results for the fixed system (inclination=34°, orientation=0°). The table below shows the monthly electricity production (E_m) and other metrics.

Month	E_g	E_m	H_g	H_m
Jan	3.35	104	3.64	113
Feb	4.31	121	4.75	133
Mar	5.17	160	5.82	180
Apr	5.30	159	6.05	182
May	5.59	173	6.44	199
Jun	5.73	172	6.76	203
Jul	6.01	186	7.15	222
Aug	6.01	186	7.18	223
Sep	5.57	167	6.59	198
Oct	4.59	142	5.27	163
Nov	3.70	111	4.15	124
Dec	3.15	97.6	3.47	107
Yearly average	4.88	148	5.61	171
Total for year		1780		2050

The data we are interested in is E_m , i.e. the monthly electricity production.

We can now gather the data for the angle optimized for maximising the energy produced over one year (kWh/year) and we can also manually set the inclination and extract the monthly production.

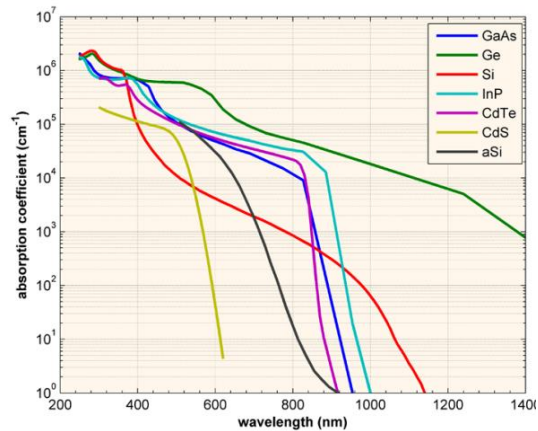
This has been done and shown in worksheet Q2.1 of the excel file and the data plotted in the chart E(theta).

So what is the increase in yield? Marked in red in the worksheet is marked the month at which production is maximised as a function of angle.

We can see that there is only an increase in c.a. 5% if we adjust the inclination of the module on a monthly basis.

2 ABSORPTION

Consider the absorption coefficient of silicon at 500nm and 800nm (check data plot on Handouts 3). Determine the minimum thickness of a slab of silicon in order to absorb 90% of the incoming radiation. Compare it with a slab of germanium.



First let us determine the absorption coefficient at 500nm and 800nm by looking at the data in the graph above.

$$\alpha_{Si}(500nm) = 1 \times 10^4 cm^{-1}$$

$$\alpha_{Si}(800nm) = 8 \times 10^2 cm^{-1}$$

$$\alpha_{Ge}(500nm) = 5 \times 10^6 cm^{-1}$$

$$\alpha_{Ge}(800nm) = 4 \times 10^4 cm^{-1}$$

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$$\alpha_{Si}(500nm) = 10^4 cm^{-1}$$

$$\alpha_{Si}(800nm) = 8 \times 10^2 cm^{-1}$$

$$\alpha_{Ge}(500nm) = 5 \times 10^6 cm^{-1}$$

$$\alpha_{Ge}(800nm) = 4 \times 10^4 cm^{-1}$$

The basic equation we want to start with is:

$$I(x) = I_0 e^{-\alpha x}$$

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This equation describes the intensity of the radiation as it passes through the absorbing material, i.e. it decreases at an exponential rate, following the Beer-Lambert law.

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What we really want to know is what is the value of x (i.e. the depth) at which 90% of the incident radiation I_0 is absorbed $A(x)$, or where only 10% is still being transmitted. i.e.:

$$\begin{aligned} I(x) &= I_0 e^{-\alpha x} \\ \therefore I_0 &= I(x) + A(x) \\ \therefore A(x) &= I_0 - I(x) \\ A(x) &= I_0 (1 - e^{-\alpha x}) \end{aligned}$$

To find the percentage, all we have to do is to divide:

$$A_{\%} = \frac{A(x)}{I_0} = (1 - e^{-\alpha x})$$

Now we can rearrange to find the value of the depth x for which $A_{\%} = 90\%$ of the incident radiation I_0 is absorbed:

$$\begin{aligned} A_{\%} &= 1 - e^{-\alpha x} \\ 1 - A_{\%} &= e^{-\alpha x} \\ \frac{\ln(1 - A_{\%})}{-\alpha} &= x \end{aligned}$$

We can now find the depth for all the values of the absorption coefficient:

$$x_{\alpha_{Si}(500nm)} = \frac{\ln(1 - 90\%)}{-\alpha_{Si}(500nm)} = \frac{\ln(1 - 90\%)}{-10^4 \text{ cm}^{-1}} = 2.3 \times 10^{-4} \text{ m} = 230 \mu\text{m}$$

$$x_{\alpha_{Si}(800nm)} = \frac{\ln(1 - 90\%)}{-\alpha_{Si}(800nm)} = \frac{\ln(1 - 90\%)}{-8 \times 10^2 \text{ cm}^{-1}} = 2.9 \times 10^{-3} \text{ m} = 2390 \mu\text{m}$$

$$x_{\alpha_{Ge}(500nm)} = \frac{\ln(1 - 90\%)}{-\alpha_{Si}(500nm)} = \frac{\ln(1 - 90\%)}{-5 \times 10^6 \text{ cm}^{-1}} \cdot x_{\alpha_{Ge}(500nm)} = \frac{\ln(1 - 90\%)}{-\alpha_{Si}(500nm)} = \frac{\ln(1 - 90\%)}{-6 \times 10^6 \text{ cm}^{-1}} = 3.8 \times 10^{-7} \text{ m} = 0.38 \mu\text{m}$$

$$x_{\alpha_{Ge}(800nm)} = \frac{\ln(1 - 90\%)}{-\alpha_{Si}(800nm)} = \frac{\ln(1 - 90\%)}{-4 \times 10^4 \text{ cm}^{-1}} = 5.8 \times 10^{-5} \text{ m} = 58 \mu\text{m}$$

As we can observe, the Ge can absorb the same amount of light as Si but in a significantly thinner layer.

3 BANDGAP

The bandgap of crystalline silicon is 1.12eV.

3.1 The energy of photons with energy below the bandgap will be lost. Considering the AM1.5 spectrum, determine the fraction of energy loss due to these photons.

We will make the simple assumption that:

All photons with energy greater than the bandgap are absorbed.

All photons with energy smaller than the bandgap are not absorbed.

The fraction of energy loss (or power loss) will be the ratio of the power not absorbed to the total incident power. Now we know the incident power, it is $P_{AM1.5} 1000W / m^2$.

In our case, we only need to account for the photons that are absorbed, i.e. those with an energy **smaller greater** than the than the bandgap. However, our data is given as Intensity as a function of wavelength, we must find what wavelength this bandgap energy corresponds to:

$$E_{photon}(\lambda) < E_g$$

$$E_g = 1.12eV$$

$$\therefore E_{photon}(\lambda) < 1.12eV$$

$$E_{photon}(\lambda) < 1.12eV$$

$$\therefore \lambda > \frac{hc}{E_g}$$

$$\lambda > \frac{6.6 \times 10^{-34} J \cdot s \times 3 \times 10^8 m/s}{1.12eV}$$

$$1eV = 1.6 \times 10^{-19} J$$

$$\therefore \lambda > \frac{6.6 \times 10^{-34} J \cdot s \times 3 \times 10^8 m/s}{1.1 \times 1.6 \times 10^{-19} J}$$

$$\lambda > 1.13 \mu m$$

So all photon with a wavelength greater that that calculated will not be absorbed. As such, we now understand what integration has to be performed:

$$I_{lost} = \int_{\lambda_{E_g}}^{\infty} I(\lambda) d\lambda$$

And because our data is discrete:

$$d\lambda \rightarrow \Delta\lambda$$

$$I(\lambda) \rightarrow I_{\Delta\lambda}(\lambda)$$

$$\therefore I_{lost} = \sum_{\lambda_{E_g}}^{\infty} I_{\Delta\lambda}(\lambda) \Delta\lambda = \Delta\lambda \sum_{\lambda_{E_g}}^{\infty} I_{\Delta\lambda}(\lambda)$$

If you now look to the excel sheet (Q3), you can see that we can now perform the summation of the intensities between the wavelength corresponding to the bandgap energy (1130nm=1.130μm) to the longest wavelength in the data (4000nm) to obtain our result of:

$$\begin{aligned} I_{lost} &= 10nm \sum_{1130nm}^{4000nm} I_{\Delta\lambda} \\ &= 188.7 \frac{W}{m^2} \end{aligned}$$

3.2 The photons with energy above the bandgap will be absorbed and contribute to the cell current. The useful energy of those photons is however limited to the bandgap energy (the rest is lost as heat). Determine the fraction of energy lost due to this inefficient energy conversion.

The first concept that needs to be understood is that the energy absorbed by an electron is that of the photon promoting the electron from the valence band to the conduction band. Thereafter, the electron loses its excess energy until it reaches the bottom of the conduction band. This process is called thermalisation.

Therefore, all electrons that are promoted to the conduction band eventually have all the same energy, that of the bandgap. As such, knowing the bandgap and the photon energy we can deduce the energy of thermalisation for absorbed photons:

$$E_{photon}(\lambda) = E_g + E_{th}$$

i.e.

$$E_{th}(\lambda) = E_{photon}(\lambda) - E_g$$

Now, the bandgap does not vary, but the photon energy does vary according to its wavelength. As such, the energy lost due to thermalisation is the difference between the photon energy and the bandgap.

To work out the power lost to thermalisation we must know what the rate of this energy is, and for that we use the photon flux which we have calculated beforehand:

As we have a photon flux, we will work out the power lost due to thermalisation.

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$$\begin{aligned}
 I_{therm}(\lambda) &= \phi(\lambda) E_{therm}(\lambda) \\
 &= \phi(\lambda) (E_{photon}(\lambda) - E_g)
 \end{aligned}$$

Now we want the total lost to thermalisation, thus:

$$\begin{aligned}
 I_{therm} &= \int_0^{\lambda_{E_g}} I_{therm}(\lambda) d\lambda \\
 &= \int_0^{\lambda_{E_g}} \phi(\lambda) (E_{photon}(\lambda) - E_g) d\lambda
 \end{aligned}$$

Because our data is not "continuous", the integration is a discrete sum:

$$\begin{aligned}
 d\lambda &\rightarrow \Delta\lambda \\
 \therefore I_{therm} &= \sum_0^{\lambda_{E_g}} \phi_\lambda(\lambda) (E_{photon}(\lambda) - E_g) \Delta\lambda = \Delta\lambda \sum_0^{\lambda_{E_g}} \phi_\lambda(\lambda) (E_{photon}(\lambda) - E_g)
 \end{aligned}$$

As before, knowing the bandgap energy we can work out the associated wavelength. Also remember that we are interested now in the photons who's energy is **greater** than the bandgap (unlike the previous exercise).

$$E_g = 1.12 eV$$

$$E_{photon}(\lambda) = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E_{photon}(\lambda)}$$

$$E_{photon}(\lambda) > E_g$$

$$\therefore \lambda < \frac{hc}{E_g}$$

$$\lambda < \frac{6.6 \times 10^{-34} J \cdot s \times 3 \times 10^8 m/s}{1.1 eV}$$

$$1 eV = 1.6 \times 10^{-19} J$$

$$\therefore \lambda < \frac{6.6 \times 10^{-34} J \cdot s \times 3 \times 10^8 m/s}{1.1 \times 1.6 \times 10^{-19} J}$$

$$\lambda < 1.13 \mu m$$

We are now in a position to perform the sum:

$$\begin{aligned} \therefore I_{therm} &= \sum_0^{1130nm} \phi(\lambda) (E_{photon}(\lambda) - 1.12eV) \times 10nm \\ &= 10nm \sum_0^{1130nm} \phi(\lambda) (E_{photon}(\lambda) - 1.72 \times 10^{-19} J) \\ &= 316.6 \frac{W}{m^2} \end{aligned}$$

3.3 Determine the ‘ideal’ efficiency of a silicon solar cell under AM1.5 irradiation (we are neglecting other very relevant loss mechanisms such as reflection and recombination).

Having calculated the power lost to:

Photons not being absorbed and

Electrons undergoing thermalisation

It follows that the power that is left over is one which we can be useful:

$$I_{total} = I_{useful} + I_{therm} + I_{lost}$$

The total power is that which is incident, and for a AM1.5 spectrum, this is by definition 1000W/m².

It therefore follows that:

$$I_{useful} = I_{total} - I_{therm} - I_{lost}$$

$$I_{total} = I_{AM1.5} = 1000W / m^2$$

$$I_{lost} = I_{lost, Eg=1.12eV} = 118.7W / m^2$$

$$I_{therm} = I_{therm, Eg=1.12eV} = 316.6W / m^2$$

$$\therefore I_{useful, Eg=1.12eV} = 565W / m^2$$

Power conversion efficiency is defined as:

$$\eta = \frac{P_{out}}{P_{in}}$$

And in our case:

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$$I_{\text{useful}} = I_{\text{useful}, E_g=1.12\text{eV}} = P_{\text{out}}$$

$$I_{\text{total}} = I_{AM1.5G} = P_{\text{in}}$$

$$\begin{aligned} \therefore \eta &= \frac{I_{\text{useful}, E_g=1.12\text{eV}}}{I_{AM1.5G}} = \frac{565\text{W} / \text{m}^2}{1000\text{W} / \text{m}^2} \\ &= 57\% \end{aligned}$$

3.4 Repeat 1-3 for a different semiconductor with a bandgap of 2.0eV. Discuss the results.

With a larger bandgap the range of wavelengths we have to work with is different:

$$E_{\text{photon}}(\lambda) < E_g$$

$$E_g = 2.0 \text{ eV}$$

$$\therefore E_{\text{photon}}(\lambda) < 2.0\text{eV}$$

$$\lambda > \frac{hc}{E_g}$$

$$\lambda > \frac{6.6 \times 10^{-34} \text{ J} \cdot \text{s} \times 3 \times 10^8 \text{ m/s}}{2.0 \times 1.6 \times 10^{-19} \text{ J}} = 0.620 \mu\text{m}$$

$$E_{\text{photon}}(\lambda) < E_g$$

$$E_g = 2.0\text{eV}$$

$$\therefore E_{\text{photon}}(\lambda) < 1.12\text{eV}$$

$$\lambda > \frac{hc}{E_g}$$

$$\lambda > \frac{6.6 \times 10^{-34} \text{ J} \cdot \text{s} \times 3 \times 10^8 \text{ m/s}}{1.1 \times 1.6 \times 10^{-19} \text{ J}}$$

$$\lambda > 0.620 \mu\text{m}$$

I.e. the range is larger, that is there is a greater range of wavelengths which are not absorbed.

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$$\begin{aligned}
 I_{lost} &= \int_{\lambda_{E_g}}^{\infty} I(\lambda) d\lambda \\
 d\lambda &\rightarrow \Delta\lambda \\
 I(\lambda) &\rightarrow I(\lambda) \\
 \\
 \therefore I_{lost} &= \sum_{\lambda_{E_g}}^{\infty} I(\lambda) \Delta\lambda = \Delta\lambda \sum_{\lambda_{E_g}}^{\infty} I(\lambda) \\
 I_{lost} &= 10nm \sum_{620nm}^{4000nm} I(\lambda) \\
 &= 639 \frac{W}{m^2}
 \end{aligned}$$

Next we work out the power lost due to thermalisation.

$$\begin{aligned}
 E_{th}(\lambda) &= E_{photon}(\lambda) - E_g \\
 \\
 I_{therm}(\lambda) &= \phi(\lambda) E_{therm}(\lambda) \\
 &= \phi(\lambda) (E_{photon}(\lambda) - E_g) \\
 \\
 I_{therm} &= \int_0^{\lambda_{E_g}} I_{therm}(\lambda) d\lambda \\
 &= \int_0^{\lambda_{E_g}} \phi(\lambda) (E_{photon}(\lambda) - E_g) d\lambda \\
 \\
 d\lambda &\rightarrow \Delta\lambda \\
 \\
 \therefore I_{therm} &= \Delta\lambda \sum_0^{\lambda_{E_g}} \phi_{\lambda}(\lambda) (E_{photon}(\lambda) - E_g)
 \end{aligned}$$

The problem is set out the same way, now we have to only modify for the bandgap.

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$$E_g = 2.0eV$$
$$= 3.2 \times 10^{-19} J$$

$$E_{\text{photon}}(\lambda) = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E_{\text{photon}}(\lambda)}$$

$$E_{\text{photon}}(\lambda) > E_g$$

$$\therefore \lambda < \frac{hc}{E_g}$$

$$\lambda < \frac{6.6 \times 10^{-34} J \cdot s \times 3 \times 10^8 m/s}{2.0eV}$$

$$1eV = 1.6 \times 10^{-19} J$$

$$\therefore \lambda < \frac{6.6 \times 10^{-34} J \cdot s \times 3 \times 10^8 m/s}{2.0 \times 1.6 \times 10^{-19} J}$$

$$\lambda < 0.62 \mu m$$

Because the bandgap is larger, the range of the spectrum which is absorbed is smaller.

$$\begin{aligned} \therefore I_{\text{therm}, E_g=2.0eV} &= \sum_0^{620nm} \phi(\lambda) (E_{\text{photon}}(\lambda) - 2.0eV) \times 10nm \\ &= 10nm \sum_0^{620nm} \phi(\lambda) (E_{\text{photon}}(\lambda) - 3.2 \times 10^{-19} J) \\ &= 74.5 \frac{W}{m^2} \end{aligned}$$

The power lost to thermalisation is noticeably smaller, because the bandgap is larger, there are fewer photons being absorbed that have an energy greater than the bandgap.

Finally we can now work out the "ideal max efficiency" for a device with a 2.0eV bandgap.

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$$I_{\text{useful}} = I_{\text{total}} - I_{\text{therm}} - I_{\text{lost}}$$

$$I_{\text{total}} = I_{AM1.5} = 1000W / m^2$$

$$I_{\text{lost}} = I_{\text{lost}, Eg=2.0eV} = 638.9W / m^2$$

$$I_{\text{therm}} = I_{\text{therm}, Eg=2.0eV} = 74.5W / m^2$$

$$\therefore I_{\text{useful}, Eg=2.0eV} = 287W / m^2$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}}$$

$$I_{\text{useful}} = I_{\text{useful}, Eg=1.12eV} = P_{\text{out}}$$

$$I_{\text{total}} = I_{AM1.5G} = P_{\text{in}}$$

$$\therefore \eta = \frac{I_{\text{useful}, Eg=2.0eV}}{I_{AM1.5G}} = \frac{287W / m^2}{1000W / m^2}$$

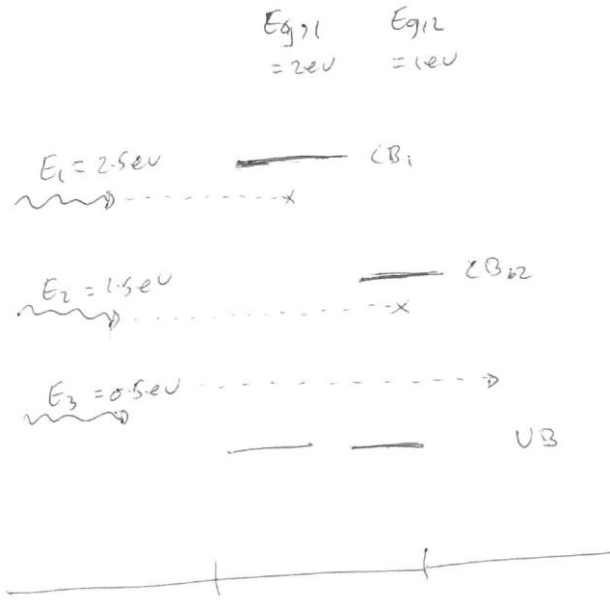
$$= 29\%$$

Comment: By almost doubling the bandgap we have reduced the power lost due to thermalisation, but have significantly increased the power which is not absorbed. As such the "ideal efficiency" decreased significantly.

*You may want to try to plot the ideal efficiency vs bandgap to find what the maximum ideal efficiency is.

3.5 Consider a solar cell made of two different layers, one with a bandgap of 2.0eV and the other one with a bandgap of 0.94eV.

3.5.1 Which layer ought to be on top? Why?



For the sake of argument, imagine a device with two layers with differing bandgaps:

$$E_{g,1} = 2eV$$

$$E_{g,2} = 1eV$$

and three incident photons of differing energies:

$$E_{ph,1} = 2.5eV$$

$$E_{ph,2} = 1.5eV$$

$$E_{ph,3} = 0.5eV$$

If the photons are incident from the left (incident on the higher bandgap layer) only the higher energy photon $E_{ph,1} = 2.5eV$ is absorbed in the first layer. The $E_{ph,2} = 1.5eV$ photon is absorbed in the second layer, and the lower energy photon is not absorbed.

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However, if the photons are incident from the right (incident on the lower bandgap layer) both $E_{ph,1} = 2.5eV$ and $E_{ph,2} = 1.5eV$ photons are absorbed in the lower bandgap layer. The lower energy photon is again not absorbed, because its energy is always smaller than any of the bandgaps.

Although the number of photons absorbed for both cases is the same (2 here) the energy lost to thermalisation is different.

In the first case the energy lost to thermalisation in each layer is:

$$E_{therm,1} = E_{ph,1} - E_{g,1} = 2.5eV - 2eV = 0.5eV$$

$$E_{therm,2} = E_{ph,2} - E_{g,2} = 1.5eV - 1eV = 0.5eV$$

In the second case with the higher energy photon being absorbed in the lower bandgap layer:

$$E_{therm,1} = 0$$

$$E_{therm,2} = (E_{ph,2} - E_{g,2}) + (E_{ph,1} - E_{g,2}) = 0.5eV + 1.5eV$$

3.5.2 Repeat 1-3 for this multijunction solar cell. Discuss the results.

Firstly we can work out the power which is not absorbed, i.e. this corresponds to the photons whose energy is smaller than both bandgaps.

$$E_{photon}(\lambda) < E_{g,2}$$

$$E_{g,2} = 0.94eV$$

$$\therefore E_{photon}(\lambda) < 0.94eV$$

$$\lambda_{Eg,2} > \frac{hc}{E_{g,2}}$$

$$\lambda_{Eg,2} > \frac{6.6 \times 10^{-34} \text{ J}\cdot\text{s} \times 3 \times 10^8 \text{ m/s}}{0.94eV} = 1.32 \mu\text{m}$$

Knowing this wavelength we can now integrate to find the intensity which is never absorbed:

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$$I_{lost} = \int_{\lambda_{E_g,2}}^{\infty} I(\lambda) d\lambda$$

$$d\lambda \rightarrow \Delta\lambda$$

$$I(\lambda) \rightarrow I_{\Delta\lambda}(\lambda)$$

$$I_{lost} = \sum_{\lambda_{E_g,2}}^{\infty} I(\lambda) \Delta\lambda = \Delta\lambda \sum_{\lambda_{E_g,2}}^{\infty} I(\lambda)$$

$$I_{lost} = 10nm \sum_{1320nm}^{4000nm} I_{\Delta\lambda}$$

$$= 114.3 \frac{W}{m^2}$$

Before we move on, we will require the wavelength corresponding to the first bandgap is (already done beforehand!):

$$E_{photon}(\lambda) < E_{g,1}$$

$$E_{g,1} = 2.0eV$$

$$\therefore E_{photon}(\lambda) < 1.12eV$$

$$\lambda_{E_g,1} > \frac{hc}{E_g} = 0.620\mu m$$

The power lost to thermalisation will be a sum of that which is lost in each individual layer. However, one must not forget that the photon flux arriving at the second layer is decreased by the amount absorbed in the first layer, i.e. we have to restrict the range of wavelengths.

For a single layer:

$$I_{therm} = \int_0^{\lambda_{E_g}} I_{therm}(\lambda) d\lambda$$

$$= \int_0^{\lambda_{E_g}} \phi(\lambda) (E_{photon}(\lambda) - E_g) d\lambda$$

However for two layers where the photon flux is incident first through the higher bandgap layer:

$$I_{therm,1} = \int_0^{\lambda_{E_{g,1}}} I_{therm}(\lambda) d\lambda$$

$$= \int_0^{\lambda_{E_{g,1}}} \phi(\lambda)(E_{photon}(\lambda) - E_{g,1}) d\lambda$$

And

$$I_{therm,2} = \int_{\lambda_{E_{g,1}}}^{\lambda_{E_{g,2}}} I_{therm}(\lambda) d\lambda$$

$$= \int_{\lambda_{E_{g,1}}}^{\lambda_{E_{g,2}}} \phi(\lambda)(E_{photon}(\lambda) - E_{g,2}) d\lambda$$

In our case, because the data is discrete:

$$I_{therm,1} = \Delta\lambda \sum_0^{\lambda_{E_{g,1}}} \phi(\lambda)(E_{photon}(\lambda) - E_{g,1})$$

$$I_{therm,2} = \Delta\lambda \sum_{\lambda_{E_{g,1}}}^{\lambda_{E_{g,2}}} \phi(\lambda)(E_{photon}(\lambda) - E_{g,2})$$

With numbers:

$$I_{therm,1} = 10nm \sum_0^{620nm} \phi(\lambda)(E_{photon}(\lambda) - 2.0eV)$$

$$= 74.5W / m^2$$

$$I_{therm,2} = 10nm \sum_{620nm}^{1320nm} \phi(\lambda)(E_{photon}(\lambda) - 0.92eV)$$

$$= 181.9W / m^2$$

$$I_{therm} = I_{therm,1} + I_{therm,2} = 256W / m^2$$

Therefore, the total power lost to thermalisation and that which is not absorbed is:

$$Lost = I_{therm} + I_{lost} = 256.7 \frac{W}{m^2} + 114.3 \frac{W}{m^2} = 371 \frac{W}{m^2}$$

The useful power is therefore:

$$I_{\text{useful}} = I_{\text{total}} - I_{\text{therm}} - I_{\text{lost}}$$

$$I_{\text{total}} = I_{AM1.5} = 1000 \frac{W}{m^2}$$

$$I_{\text{lost}} = 114.3 \frac{W}{m^2}$$

$$I_{\text{therm}} = 256.7 \frac{W}{m^2}$$

$$\therefore I_{\text{useful}} = 629 \frac{W}{m^2}$$

i.e.

$$\eta = \frac{I_{\text{useful}}}{I_{\text{total}}} = \frac{629 W / m^2}{1000 W / m^2} = 63\%$$

4 TRANSPARENT MODULES

For some applications (e.g. solar windows) it might be interesting to allow the visible part of the spectrum to get through the solar cell, only capturing the infrared part of the spectrum. Determine the ideal efficiency of a transparent solar module.

This exercise is the same as the exercise beforehand, except we have to choose a bandgap such visible light is not absorbed. The visible spectrum can be defined as light with wavelengths:

$$400 < \lambda < 700nm$$

If we convert this to photon energy:

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$$400 < \lambda < 700nm$$

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

$$E_{\lambda=400nm} = 4.95 \times 10^{-19} J = 3.1eV$$

$$E_{\lambda=700nm} = 2.83 \times 10^{-19} J = 1.8eV$$

$$\therefore 3.1eV > \lambda > 1.8eV$$

I.e., to let visible light through between 3.1eV and 1.8eV, the bandgap of the transparent solar cell has to be greater than 3.1eV, and this corresponds to a wavelength of 400nm.

We can now repeat the exercise for determining the ideal efficiency:

1 - determine incident radiation which is not absorbed:

$$I_{\text{lost}} = \int_{\lambda_{E_g}}^{\infty} I(\lambda) d\lambda$$

$$\lambda_{E_g=3.1eV} = 400nm$$

$$I_{\text{lost}} = \int_{400nm}^{\infty} I(\lambda) d\lambda = \Delta\lambda \sum_{\lambda_{E_g}}^{\infty} I_{\Delta\lambda}(\lambda)$$

$$\begin{aligned} I_{\text{lost}} &= 10nm \sum_{400nm}^{4000nm} I_{\Delta\lambda} \\ &= 953.7 \frac{W}{m^2} \end{aligned}$$

And the power lost to thermalisation:

$$\begin{aligned} I_{\text{therm}} &= \int_0^{\lambda_{E_g}} I_{\text{therm}}(\lambda) d\lambda \\ &= \int_0^{\lambda_{E_g}} \phi(\lambda) (E_{\text{photon}}(\lambda) - E_g) d\lambda \end{aligned}$$

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$$\begin{aligned}\therefore I_{therm} &= \sum_0^{1130nm} \phi(\lambda)(E_{photon}(\lambda) - 1.12eV) \times 10nm \\ &= 10nm \sum_0^{1130nm} \phi(\lambda)(E_{photon}(\lambda) - 1.72 \times 10^{-19} J) \\ &= 4.4 \frac{W}{m^2}\end{aligned}$$

Thus the ideal efficiency is:

$$\eta = \frac{I_{useful}}{I_{total}} = \frac{4.4W / m^2}{1000W / m^2} = 0.04\%$$

5 CURRENT

Considering the AM1.5 spectrum, determine the maximum current density (mA/cm²) of a silicon solar cell (assuming that every useful photon will produce an electron-hole pair).

Firstly, every usefull photon is the total number of photons absorbed, and that is all photons with an energy greater than the bandgap are absorbed.

$$\phi_{abs} = \phi_{e^-} = \phi_{h^+}$$

Thus:

$$\phi_{abs} = \int_0^{\lambda_{eg}} \phi(\lambda) d\lambda$$

For silicon, with a bandgap of 1.12eV, this means that all photon with an energy greater than 1.12eV are absorbed, or with a wavelength below 1105nm.

$$\phi_{abs, Si} = \int_0^{1105nm} \phi(\lambda) d\lambda$$

Or using our data:

$$\begin{aligned}\phi_{abs, Si} &= 10nm \sum_{400}^{1110nm} \phi(\lambda) \\ &= 2.73 \times 10^{20} \frac{1}{m^2 s}\end{aligned}$$

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Or:

$$\phi_{e^-} = \phi_{h^+} = 2.73 \times 10^{20} \frac{1}{m^2 s}$$

This flow electrons and holes can be converted to a current because this is effectively a flow of charge. By the units of current (Amp.)

$$[A] = \frac{C}{s}$$

And each electron and hole has q charge:

$$\begin{aligned} J &= q\phi_{e^-} \\ &= 1.6 \times 10^{-19} C \times 2.73 \times 10^{20} \frac{1}{m^2 s} \\ &= 43.7 \frac{C}{m^2 s} \\ &= 43.7 \frac{A}{m^2} \end{aligned}$$

6 FILL FACTOR

If a 100cm² solar cell has a rated efficiency of 15%, an open circuit voltage of 0.6V and a short current of 3.33A, what is the fill factor?

Start with a definition of fill factor and work through with what we are given:

$$ff = \frac{i_{mpp} V_{mpp}}{i_{sc} V_{oc}}$$

$$\begin{aligned} P_{max} &= i_{mpp} V_{mpp} \\ \therefore ff &= \frac{P_{max}}{i_{sc} V_{oc}} \end{aligned}$$

$$i_{sc} = 3.33 \frac{A}{100cm^2}$$

$$V_{oc} = 0.6V$$

$$\therefore ff = \frac{P_{max}}{3.33 \frac{A}{100cm^2} \times 0.6V}$$

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Now we have to think about how we can calculate P_{max} . Remember that the solar cell is rated as 15% efficient, and this is typically a rating under AM1.5 conditions, i.e. $1000W/m^2$. Therefore:

$$\begin{aligned}\eta &= \frac{P_{out}}{P_{in}} \\ \eta &= 15\% \\ P_{out} &= P_{max} \\ P_{in} &= P_{AM1.5} = 1000W/m^2 \\ \\ \therefore P_{max} &= \eta P_{AM1.5} \\ &= 15\% \times 1000W/m^2 \\ &= 150W/m^2\end{aligned}$$

And remembering to take into account the actual area of the solar cell, the maximum power output is:

$$P_{max} = 150W/m^2 = 1.5 \frac{W}{100cm^2}$$

Thus:

$$\begin{aligned}I_{sc} &\equiv I(V=0) = I_L \\ ff &= \frac{P_{max}}{3.33 \frac{A}{100cm^2} \times 0.6V} = \frac{1.5 \frac{W}{100cm^2}}{3.33 \frac{A}{100cm^2} \times 0.6V} \\ &= 75\%\end{aligned}$$

7 DIODE EQUATION

7.1 Looking at the diode equation $I = I_L - I_s \left\{ e^{\frac{q}{k_b T} V} - 1 \right\}$ write out the equations describing

7.1.1 in the dark,

$$\begin{aligned}I &= I_L - I_s \left\{ e^{\frac{q}{k_b T} V} - 1 \right\} \\ I_L &= 0 \\ \therefore I &= -I_s \left\{ e^{\frac{q}{k_b T} V} - 1 \right\}\end{aligned}$$

7.1.2 illuminated short circuit conditions, i.e. no applied voltage, and

$$I = I_L - I_s \left\{ e^{\frac{q}{k_B T} V} - 1 \right\}$$
$$V = 0$$
$$\therefore I(V=0) = I_L - I_s \left\{ e^0 - 1 \right\}$$
$$I(V=0) = I_L$$

7.1.3 illuminated open circuit conditions i.e. no current flow.

$$I = I_L - I_s \left\{ e^{\frac{q}{k_B T} V} - 1 \right\}$$
$$I = 0$$
$$\therefore 0 = I_L - I_s \left\{ e^{\frac{q}{k_B T} V} - 1 \right\}$$
$$I_L = I_s \left\{ e^{\frac{q}{k_B T} V} - 1 \right\}$$

**7.2 Determine the expression describing the short circuit current I_{sc} and open circuit voltage V_{oc} .
What do these expressions tell us?**

When there is no applied voltage the diode equation is reduced to:

$$I(V=0) = I_L$$

What this tells us is that the short circuit current (when $V=0$) is equal and directly proportional to the photocurrent generated by the absorption of light.

When there is no current flow, the diode equation reduces to:

$$I_L = I_s \left\{ e^{\frac{q}{k_B T} V} - 1 \right\}$$

What this tells us is that the total recombination current within the cell is equal to the current generated by light absorption. We can rearrange the expression to obtain the voltage at which this is the case is open circuit voltage:

$$I_L = I_s \left\{ e^{\frac{q}{k_B T} V(I=0)} - 1 \right\}$$

$$I_L = I_s e^{\frac{q}{k_B T} V(I=0)} - I_s$$

$$\ln \frac{I_L + I_s}{I_s} = \frac{q}{k_B T} V(I=0)$$

$$\therefore V(I=0) = V_{oc} = \frac{k_B T}{q} \ln \frac{I_L + I_s}{I_s}$$

$$I_L \gg I_s$$

$$V_{oc} = \frac{k_B T}{q} \ln \frac{I_L}{I_s}$$

7.3 Considering the solar cell in question 4, and the diode equation, what does this tell us about the magnitude of the saturation current for the specific solar cell?

The V_{oc} of the solar cell is 0.6V. As such:

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$$V_{oc} = \frac{k_B T}{q} \ln \frac{I_L}{I_s}$$

$$V_{oc} \frac{q}{k_B T} = \ln \frac{I_L}{I_s}$$

$$V_{oc} = 0.6V = 0.6 \frac{J}{C}$$

$$T = 300K$$

$$q = 1.6 \times 10^{-19} C$$

$$k_B = 1.38 \times 10^{-23} JK^{-1}$$

$$\frac{k_B T}{q} = 0.026 \frac{J}{C} \approx \frac{1}{40} V$$

$$\frac{q}{k_B T} = 40 \frac{1}{V}$$

$$\therefore V_{oc} \frac{q}{k_B T} = 0.6V \times 40 \frac{1}{V} = 24$$

$$\therefore \ln \frac{I_L}{I_s} \approx 24$$

$$\therefore I_L \approx I_s e^{24}$$

Given that we know all the variables except for the saturation current, we can calculate it:

$$I_L \exp\left\{-V_{oc} \frac{q}{k_B T}\right\} = I_s$$

$$I_L = I_{sc} = 3.33A \frac{A}{cm^2} = 33.3 \frac{mA}{cm^2}$$

$$V_{oc} = 0.6V$$

$$\frac{q}{k_B T} = 40V^{-1}$$

$$I_s = 33.3 \frac{mA}{cm^2} \times \exp\{-0.6V \times 40V^{-1}\}$$

$$= 1.3 \times 10^{-9} \frac{mA}{cm^2}$$

7.4 In ideal conditions the current produced by a solar cell at short circuit is linear with the light intensity. Assuming these conditions, how does V_{oc} vary with light intensity?

i.e.

$$I_L \propto P_{inc}$$

Let us think of a multiplying factor which is by how much the intensity of the incident radiation varies and how this changes the open circuit voltage.

$$x = \frac{P'_{inc}}{P_{inc}}$$

$$V_{oc} = \frac{k_B T}{q} \ln \frac{I_L}{I_s}$$

$$\therefore V'_{oc} = \frac{k_B T}{q} \ln \frac{x I_L}{I_s}$$

$$= \frac{k_B T}{q} \left(\ln \frac{I_L}{I_s} + \ln x \right) = \frac{k_B T}{q} \ln \frac{I_L}{I_s} + \frac{k_B T}{q} \ln x$$

i.e.

$$V'_{oc} = V_{oc} + \frac{k_B T}{q} \ln x$$

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Thus, e.g. if I have a 10 fold increase in the incident light intensity, and assume that the photocurrent also increases 10 fold then:

$$P'_{inc} = 10P_{inc}$$

$$\therefore x = \frac{P'_{inc}}{P_{inc}} = 10$$

$$V_{oc} = \frac{k_B T}{q} \ln \frac{I_L}{I_s}$$

$$\therefore = \frac{k_B T}{q} \ln \frac{xI_L}{I_s}$$

$$V'_{oc} = V_{oc} + \frac{k_B T}{q} \ln x$$

$$= V_{oc} + \frac{1}{40} V^{-1} \ln 10$$

$$= V_{oc} + 0.058V$$

i.e., the voltage "only" increases by 0.058V or 58mV per decade increase in the incident light intensity.